Register Number	**Class**	**Name**

PEICAI SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2010
SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS (4016/02)
PAPER 2

DATE: 14 SEPTEMBER 2010 (TUESDAY)
DURATION: 2 HOURS 30 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **10** printed pages and **0** blank page.

Set by: Ms June Yeo
Vetted by: Ms Lee Chiou Kwei

Mathematical Formulae

Compound Interest

$$\text{Total amount } = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1 **(a)** Evaluate $\sqrt{\dfrac{2.471}{0.4328 \times 73.54}}$, giving your answer correct to 3 significant figures. [1]

(b) Solve the equations

(i) $(1-3x)^2 = 16$, [2]

(ii) $p(p-3) = 2p+6$. [3]

(c) Express as a single fraction in its simplest form $\dfrac{c^2(a^2+6ab-7b^2)}{6c(a^2-b^2)}$. [3]

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2 **(a)** A cyclist rode a distance of 42 km at an average speed of y km/h. Write down an expression for the time, in hours, that he took for the journey. [1]

(b) He returned by the same route but his average speed was 2 km/h less. Write down an expression for the time, in hours, that he took for the return journey. [1]

(c) Given that the difference between the two times taken was 1 hour 25 minutes, form an equation in y and show that it reduces to $17y^2 - 34y - 1008 = 0$. [3]

(d) Solve this equation, giving your answers correct to 1 decimal place. [3]

(e) Hence find the time, correct to the nearest 5 minutes, for the return journey. [1]

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3 **(a)** The diagram shows the cross-section of a solid which consists of a circle overlapping a trapezium at A and B. The circle, centre O, has radius 1 m. $ABCD$ is a trapezium in which AB is parallel to DC. N is the point on DC such that angle $AND = 90°$. $AD = BC = 3$ m and $AN = 2.5$ m.
Given that arc $AB = 1.2$ m, calculate

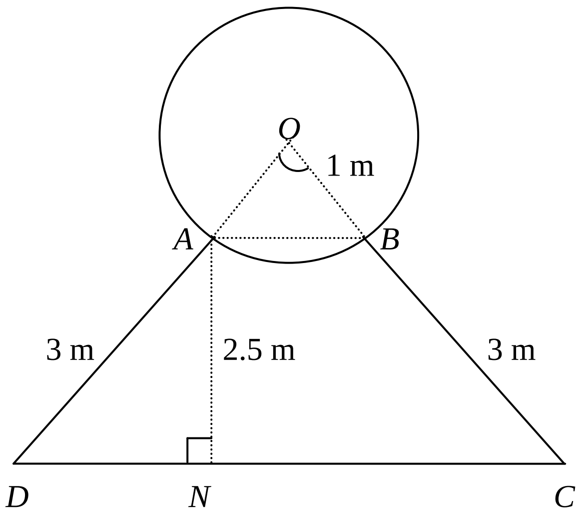
(i) the acute angle AOB in radians, [1]

(ii) AB, [2]

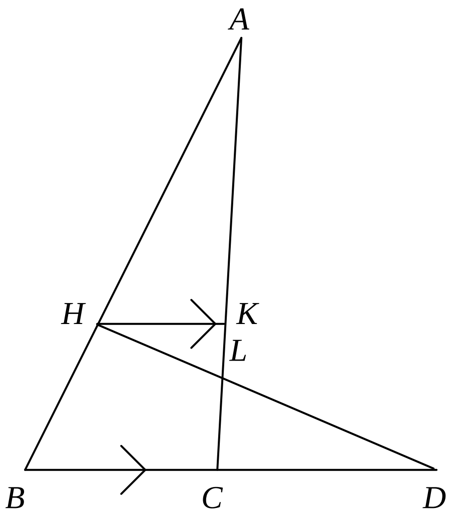
(iii) the area of the triangle AOB, [2]

(iv) the area of the cross-section of the solid. [5]

(b) The solid is made of concrete which has a density of 2.3 g / cm^3.
The thickness of the solid is 30 cm. Find the total mass of the solid, expressing your answer in kg. [2]



4 **(a)** In the diagram, $AH = BD$, $HK = HB$ and HK is parallel to BD.



(i) Name the triangle which is congruent to ΔAHK. [1]

(ii) Prove that ΔAHL is similar to ΔDCL. [2]

(iii) It is also given that $AH = 9$ cm, $HL = 3$ cm and $CD = 5$ cm.
Calculate CL. [1]

(b) There are 25 children in a class. Of these, 12 are in the School Play and 18 are in the School Choir. It is given that ε = {children in the class}, P = {children in the School Play}, and C = {children in the School Choir}.

(i) Draw a venn diagram to represent the above information. [1]

(ii) Find the smallest possible value of $n(P \cap C)$. [1]

(iii) Find $n(P')$. [1]

(iv) Express in set notation {children who are neither in the School Play nor the School Choir}. [1]

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5 The equation of a straight line is $5y + 4x = -8$. Calculate

(a) the gradient of the line, [1]

(b) the coordinates of the point where the line crosses the x-axis, [1]

(c) the coordinates of the point at which the line intersects the line $x = 7$, [1]

(d) the equation of the line, parallel to the given line, which passes through the point (2, 5). [2]

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6 Part of a pattern of numbers is shown in the table below.

	Column 1	**Column 2**	**Column 3**
Row 1	97	2	1
Row 2	94	8	8
Row 3	91	18	27
Row 4	88	32	64
Row 5	x	y	z
Row n	p	q	r

(a) Study the pattern and write down the value of x, the value of y and the value of z. [3]

(b) Express, as simply as possible, in terms of n, the value of p, the value of q and the value of r. [3]

(c) In which row does the first negative term appear in Column 1? [1]

(d) In which row does the first term greater than 50 000 appear in Column 2? [1]

(e) In which row does the first term greater than 2×10^8 appear in Column 3? [1]

7 15 students from Class 3A went on an Elements of Business Skills (EBS) cum Service Learning trip in Batam last July. The ferry left Singapore at 0830 on Tuesday, and they arrived at Batam Center Ferry Terminal at local time 0815 on the same day.

(a) If Singapore time is ahead of Batam by an hour, calculate how long their journey was. [1]

(b) One student, Amalia promised her mother to call home an hour after she had arrived in Batam. What time did her mother receive her call in Singapore? [1]

(c) Amalia called home from her handphone, which charges a rate of 20 cents for the first 5 minutes and subsequent usage at 10 cents per minute. If her call lasted 18 minutes, calculate the cost of this call. [1]

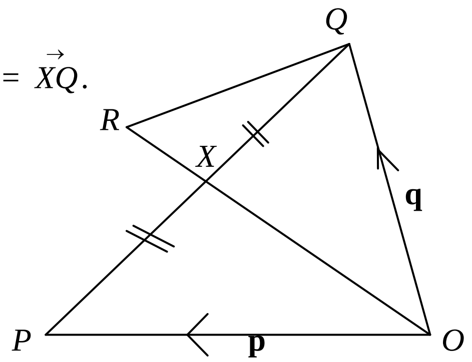
(d) In Batam, Amalia bought souvenir pens for 37 relatives and friends. If the pens were sold in packets of 8, how many packets must Amalia buy and how many extras would she have? [1]

Amalia brought a total of 500 000 Indonesian Rupiah (IDR) for her trip. She spent $\frac{1}{5}$ of it buying groceries for a village family in Batam, and $\frac{1}{2}$ of the remainder on sightseeing and shopping. She changed what she had left to Singapore dollars when she went home.

(e) Calculate in IDR, the amount Amalia spent on sightseeing and shopping. [2]

(f) Calculate the amount of Singapore dollars Amalia was left with, given that the exchange rate was S$1 to 6,614 Indonesian Rupiah (IDR). [2]

8 In the diagram, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{PX} = \overrightarrow{XQ}$.



(a) Express in terms of **p** and **q**

(i) $\overrightarrow{PX}$, [1]

(ii) $\overrightarrow{OX}$. [1]

(b) Given that $\overrightarrow{OR} = \frac{3}{4}\mathbf{p} + \frac{3}{4}\mathbf{q}$, write down the numerical value of the ratio $\frac{RX}{XO}$. [2]

(c) $\overrightarrow{QR}$ is produced to point Y where $\overrightarrow{QY} = h\overrightarrow{QR}$. Obtain expression for $\overrightarrow{QR}$ and $\overrightarrow{QY}$ and hence prove that $\overrightarrow{OY} = \frac{3h}{4}\mathbf{p} + \left(1 - \frac{h}{4}\right)\mathbf{q}$. [3]

(d) Given also that $\overrightarrow{OY} = k\overrightarrow{OP}$, form an equation involving **p**, **q**, h and k and use it to find the value of h and k. [3]

(e) **Hence**, deduce and state the numerical value of the ratio $\frac{YP}{PO}$. [1]

9 ***Answer the whole of this question on a sheet of graph paper.***

The variables x and y are connected by the equation $y = 2x^2 - 5x - 3$ and some corresponding values are given in the following table.

x	–2	–1	0	$\frac{1}{2}$	1	2	3	4	5
y	a	4	–3	b	–6	–5	0	9	22

(a) Calculate the values of a and b. [2]

(b) Taking 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw the graph $y = 2x^2 - 5x - 3$ for the range $-2 \le x \le 5$. [3]

(c) From your graph, find the values of x when $y = 5$. [1]

(d) Write down the equation of the line of symmetry of the graph. [1]

(e) Find, by drawing a tangent, the gradient of the graph at the point where $x = 2$. [2]

(f) By drawing a suitable straight line on the same axes, use your graph to find the solutions of the equation $2x^2 - 8x = 7$. [3]

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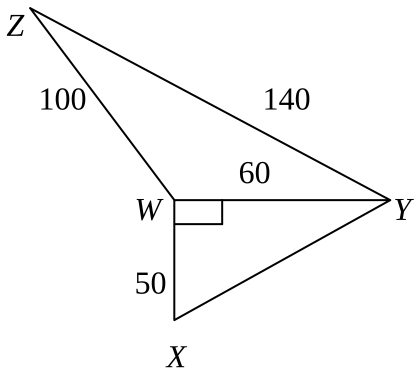
10 W, X, Y and Z are four points on horizontal ground. $YZ = 140$ m, $WZ = 100$ m, X is 50 m due south of W and Y is 60 m due east of W. Calculate

(a) angle WXY, [2]

(b) angle ZWY, [3]

(c) the bearing of W from Z, [2]

A vertical mast stands at the point W and the angle of elevation of the top of this mast from each of the points X, Y and Z is known. Given that the smallest of these angles of elevation is 12°, calculate the height of the mast in metres, giving your answer correct to three significant figures. [2]



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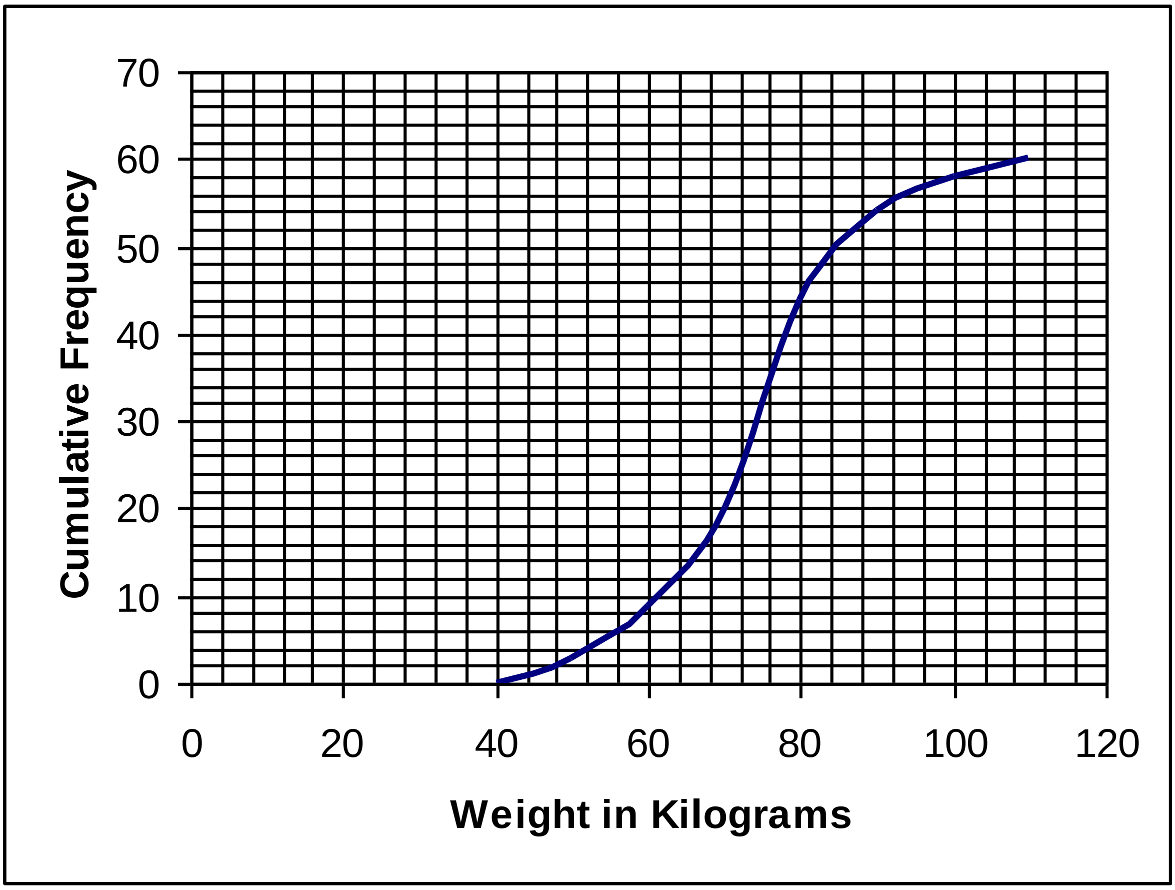
11 **(a)** The graph is a cumulative frequency curve showing the distribution of the weights, in kilograms, of the 60 members of a sports club. Use the curve to estimate

(i) the median weight, [1]

(ii) the interquartile range, [2]

(iii) One member is chosen at random from the 60. Using your graph, estimate the probability that his weight is less than or equal to 54 kg. [1]



(b) The following table gives the frequency distribution of marks achieved by 600 candidates in an English examination.

Mark	$0 \le x \le 20$	$20 < x \le 40$	$40 < x \le 60$	$60 < x \le 80$	$80 < x \le 100$
English	20	100	330	110	40

Find

(i) the mean mark, [2]

(ii) the standard deviation of the marks [2]

achieved by the 600 candidates.

~ End of Paper ~

Peicai Secondary School
Math Department
Secondary 4 Express / 5 Normal Academic
Mathematics Paper 2 – Mark Scheme

1 (a) $\sqrt{\dfrac{2.471}{0.4328\times 73.54}} = 0.279\ \left(3\ \textit{significant figures}\right)\ [A1]$

(b) (i)

$$\left(1-3x\right)^2 = 16$$
$$1-3x=\pm 4\ [M1]$$
$$1-3x=-4,\ 1-3x=4$$
$$x=\frac{5}{3},\ -1\ [A1]$$

(ii)

$$p\left(p-3\right)=2p+6$$
$$p^2-3p=2p+6$$
$$p^2-5p-6=0\ [M1]$$
$$\left(p-6\right)\left(p+1\right)=0\ [M1]$$
$$p=-1,\ 6\ [A1]$$

(c)

$$\frac{c^2\left(a^2+6ab-7b^2\right)}{6c\left(a^2-b^2\right)}$$
$$=\frac{c\left(a-b\right)\left(a+7b\right)}{6\left(a-b\right)\left(a+b\right)}\ [M2]$$
$$=\frac{c\left(a+7b\right)}{6\left(a+b\right)}\ [A1]$$

2 (a) $\left(\dfrac{42}{y}\right)\ h\ [A1]$

(b) $\left(\dfrac{42}{y-2}\right)\ h\ [A1]$

(c)

$$\frac{42}{y-2}-\frac{42}{y}=\frac{85}{60}\ [M1]$$

$$\frac{42}{y-2}-\frac{42}{y}=\frac{17}{12}$$

$$\frac{42y-42(y-2)}{y(y-2)}=\frac{17}{12}\ [M1]$$

$$12(42y-42y+84)=17y(y-2)$$

$$1008=17y^2-34y$$

$$17y^2-34y-1008=0\ (shown)\ [A1]$$

(d)

$$17y^2-34y-1008=0$$

$$y=\frac{-(-34)\pm\sqrt{(-34)^2-4(17)(-1008)}}{2(17)}$$

$$b^2-4ac=69700\ [M1],\ 2a=34\ [M1]$$

$$y=\frac{34\pm\sqrt{69700}}{34}$$

$$=8.8,\ -6.8\ (rejected)\ (1\ decimal\ place)\ [A1]$$

(e)

$$\frac{42}{8.8-2}$$

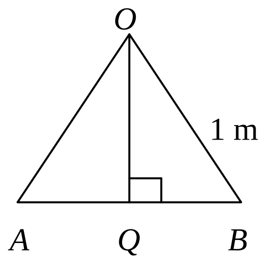
$$=370\ \text{minutes}\ (nearest\ 5\ \text{minutes})\ [A1]$$

3 (a) (i)

$$1(\angle AOB)=1.2$$

$$\angle AOB=1.2\ radians\ [A1]$$

(ii)



$\angle QOB = 0.6 \ radian$

$\sin \angle QOB = \dfrac{BQ}{OB}$

$\sin 0.6 = \dfrac{BQ}{1} \ [M1]$

$BQ = 0.5646 \ m \ \left(4 \ significant \ \ figures\right)$

$AB = 2 \times 0.5646$

$= 1.13 \ m \ \left(3 \ significant \ \ figures\right) \ [A1]$

$alternative \ method: \ \cos ine \ rule$

$AB^2 = 1^2 + 1^2 - 2\left(1\right)\left(1\right)\cos 1.2 \ [M1]$

$AB = 1.13 \ m \ \left(3 \ significant \ \ figures\right) \ [A1]$

(iii)

$\dfrac{1}{2}\left(OA\right)\left(OB\right)\sin \angle AOB$

$= \dfrac{1}{2}\sin 1.2 \ [M1]$

$= 0.466 \ m^2 \ \left(3 \ significant \ \ figures\right) \ [A1]$

(iv)

$area \ of \ \ \sec tor \ OAB$

$= \dfrac{1}{2}\left(OA\right)\left(OB\right)\angle AOB$

$= \dfrac{1}{2}\left(1.2\right)$

$= 0.6 \ m^2 \ [M1]$

$area \ of \ \ \min or \ segment \ AB$

$= 0.6 - 0.466$

$= 0.134 \ m^2 \ [A1]$

$DN^2 + 2.5^2 = 3^2$

$DN = \sqrt{2.75}\ m$

$area\ of\ trapezium\ ABCD$

$= \frac{1}{2}(2.5)\left(2\sqrt{2.75} + 2(1.13)\right)$

$= 6.971\ m^2\ \left(4\ significant\ \ figures\right)\ [M1]$

$area\ of\ \ \mathrm{m}ajor\ segment\ AB$

$= \pi(1)^2 - 0.134$

$= 3.008\ m^2\ \left(4\ significant\ \ figures\right)\ [M1]$

$area\ of\ cross-\sec tion = 3.008 + 6.971$

$= 9.979\ m^2\ [A1]$

(b) volume of solid
= 9.979 $\times$ 0.3
= 2.9937 m^3 [M1]
= 2 993 700 cm^3
1 cm^3 --- 2.3 g
2 993 700 cm^3 --- 6 885 510 g = 6 885.51 kg [A1]

4 (a) (i) ΔDBH [A1]

(ii)

$\angle ALH = \angle CLD\ \left(vert.\ opp.\ \angle s\right)$

$\angle HAK = \angle CDL\ \left(\Delta AHK\ congruent\ to\ \Delta DBH\ \ /\ \ alt.\ \angle s\right)$

$hence\ \angle AHL = \angle DCL$

$above\ steps:\ [M1]$

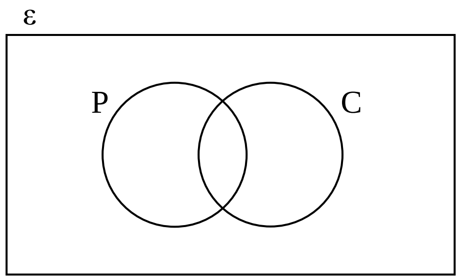
3 equal pairs of corresponding angles [A1]

(iii)

$\frac{CL}{3} = \frac{5}{9}\ [M1]$

$CL = \frac{5}{3}\ cm\ [A1]$

(b) (i)



[A1]

(ii) 5 [A1]

(iii) 13 [A1]

(iv) $(P \cup C)'$ [A1]

5 (a) $5y + 4x = -8$
$5y = -4x - 8$
$y = -0.8x - 1.6$
gradient = –0.8 [A1]

(b) $y = 0$: $4x = -8$
$x = -2$ [A1]

(c) $x = 7$: $5y + 4(7) = -8$
$y = -7.2$ [A1]

(d) $y = -0.8x + c$
(2, 5): $5 = -0.8(2) + c$
$c = 6.6$
above steps: [M1]
$y = -0.8x + 6.6$ [A1]

6 (a) $x = 3$ [A1], $y = 50$ [A1], $z = 125$ [A1]

(b) $p = 97 - 3(n - 1) = 97 - 3n + 3 = 100 - 3n$ [A1], $q = 2n^2$ [A1], $r = n^3$ [A1]

(c) $n = 34$ [A1]

(d) $2n^2 > 50\,000$; $n^2 > 25\,000$; $n > 158.1$ (4 significant figures), hence $n = 159$ [A1]

(e) $n^3 > 2 \times 10^8$; $n > 584.8$ (4 significant figures), hence $n = 585$ [A1]

7 (a) 45 minutes [A1]

(b) 1015 [A1]

(c) $20 + (10 \times 13) = 150$ cents / \$1.50 [A1]

(d) $37 \div 8 = 4.625$; She must buy 5 packets; 3 extras. [A1]

(e)

$buying\ \ groceries\ \ for\ a\ village\ \ family: \ \frac{1}{5} \times 500000 = 100000\ \ IDR$

$remainder = 500000 - 100000 = 400000\ \ IDR$

$above\ \ steps: \ M1$

$amount\ \ spent\ \ on\ \ sightseeing\ \ \&\ : \ \frac{1}{2}\ \ of\ \ remainder = 200000\ \ IDR\ [A1]$

(f) amount left = 500 000 – 100 000 – 200 000 = 200 000 IDR [M1]
6,614 IDR = S$1
200 000 IDR = S$30.24 [A1]

8 (a) (i)

$\overrightarrow{PQ} = -\underset{\sim}{p} + \underset{\sim}{q}$

$\overrightarrow{PX} = \frac{1}{2}\left(-\underset{\sim}{p} + \underset{\sim}{q}\right)\ [A1]$

(ii)

$\overrightarrow{OX} = \underset{\sim}{p} + \frac{1}{2}\left(-\underset{\sim}{p} + \underset{\sim}{q}\right)$

$= \underset{\sim}{p} - \frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}$

$= \frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\ [A1]$

(b)

$\overrightarrow{XR} = \overrightarrow{OR} - \overrightarrow{OX}$

$= \frac{3}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q} - \left(\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{q}\right)$

$= \frac{3}{4}\underset{\sim}{p} + \frac{3}{4}\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{p} - \frac{1}{2}\underset{\sim}{q}$

$= \frac{1}{4}\underset{\sim}{p} + \frac{1}{4}\underset{\sim}{q}$

$= \frac{1}{4}\left(\underset{\sim}{p} + \underset{\sim}{q}\right)\ [M1]$

$\frac{RX}{XO} = \frac{\frac{1}{4}}{\frac{1}{2}}$

$= 0.5\ [A1]$

(c)

$$\overrightarrow{QR} = \overrightarrow{QX} + \overrightarrow{XR}$$
$$= -\frac{1}{2}\left(-\underset{\sim}{p} + \underset{\sim}{q}\right) + \frac{1}{4}\left(\underset{\sim}{p} + \underset{\sim}{q}\right)$$
$$= \frac{1}{2}\underset{\sim}{p} - \frac{1}{2}\underset{\sim}{q} + \frac{1}{4}\underset{\sim}{p} + \frac{1}{4}\underset{\sim}{q}$$
$$= \frac{3}{4}\underset{\sim}{p} - \frac{1}{4}\underset{\sim}{q}$$
$$= \frac{1}{4}\left(3\underset{\sim}{p} - \underset{\sim}{q}\right) \; [A1]$$
$$\overrightarrow{QY} = h\overrightarrow{QR}$$
$$= \frac{h}{4}\left(3\underset{\sim}{p} - \underset{\sim}{q}\right) \; [A1]$$
$$\overrightarrow{OY} = \overrightarrow{OQ} + \overrightarrow{QY}$$
$$= \underset{\sim}{q} + \frac{h}{4}\left(3\underset{\sim}{p} - \underset{\sim}{q}\right)$$
$$= \frac{3h}{4}\underset{\sim}{p} + \left(1 - \frac{h}{4}\right)\underset{\sim}{q} \; \left(proven\right) \; [A1]$$

(d)

$$\frac{3h}{4}\underset{\sim}{p} + \left(1 - \frac{h}{4}\right)\underset{\sim}{q} = k\underset{\sim}{p}$$
$$1 - \frac{h}{4} = 0; \; \frac{3h}{4} = k \; [M1]$$
$$\frac{3(4)}{4} = k$$
$$h = 4 \; [A1]; \; k = 3 \; [A1]$$

(e)

$$\overrightarrow{OY} = 3\underset{\sim}{p}$$
$$\overrightarrow{YP} = \overrightarrow{YO} + \overrightarrow{OP} = -2\underset{\sim}{p}$$
$$\frac{YP}{PO} = \frac{-2}{-1} = 2 \; [A1]$$

$$x = -2, \; y = a:$$
$$a = 2(-2)^2 - 5(-2) - 3$$
$$= 15 \; [A1]$$

9 (a)

$$x = \frac{1}{2}, \; y = b:$$
$$b = 2\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) - 3$$
$$= -5 \; [A1]$$

10 (a)

$$\tan\angle WXY = \frac{60}{50} \ [M1]$$

$$\angle WXY = 50.2^\circ \ [A1]$$

(b)

$$140^2 = 100^2 + 60^2 - 2(100)(60)\cos\angle ZWY \ [M2]$$

$$\angle ZWY = 120^\circ \ [A1]$$

(c)

$$\angle ZWX = 360^\circ - 120^\circ - 90^\circ$$

$$= 150^\circ \ [M1]$$

$$bearing\ of\ W\ \ from\ Z = 150^\circ \ (alt.\ \angle s) \ [A1]$$

$$\tan 12^\circ = \frac{WMast}{100} \ [M1]$$

$$WMast = 21.3\ m\ \left(3\ significant\ \ figures\right) \ [A1]$$

11 (a) (i) 74 kg [A1]

(ii) 80 – 66 [M1] = 14 kg [A1]

(iii) $\frac{5}{60} = \frac{1}{12}$ [A1]

(b) (i) mean mark

$$= \frac{(10\times 20)+(30\times 100)+(50\times 330)+(70\times 110)+(90\times 40)}{600} \ [M1]$$

$$= 51\frac{2}{3} \ [A1]$$

(ii) standard deviation

$$= \sqrt{\frac{1780000}{600} - \left(51\frac{2}{3}\right)^2} \ [M1]$$

$$= 17.2 \ [A1]$$